مركز الفجر للخدمات العلمية
٠١٠٠٢٣٦٠٠٦

Electromagnetic

Lecture (4)

Chapter (1)
Part (3)
"Coulomb's Law"

2nd year
communications

مركز الفجر للخدمات العلمية
٠١٠٠٢٣٦٠٠٦

Lecture (9) :

part (3)

Chapter (1)

Sheet (2)

Q1 Infinite line : $\rho_L = 2\ nC/m$ (x-axis)

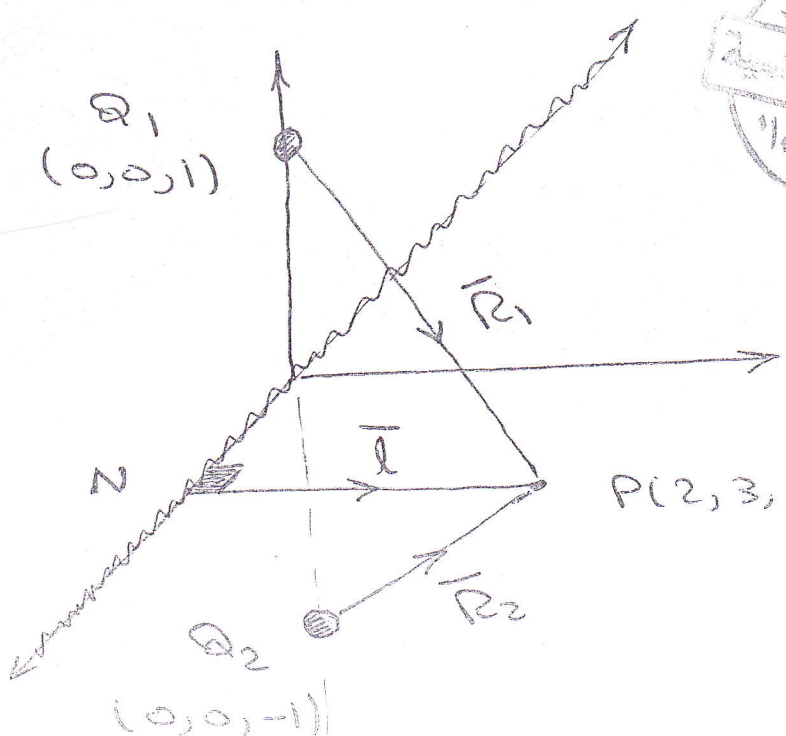
2 charges : $Q_1 = 8\ nC$ $(0,0,1)$

$Q_2 = 8\ nC$ $(0,0,-1)$

(a) Find $\bar{E}_P$ at point $P(2,3,-4)$

(b) Find $\rho_L = ?$ that makes $\bar{E}$ at point $(0,0,3) = 0$.

Solution



مركز الفجر
للخدمات العلمية
٠١٠٠٩٢٢٦٠٠٦

(a) $\bar{E}_P = \bar{E}_\ell + \bar{E}_{Q_1} + \bar{E}_{Q_2}$

Line = { infinite, ρ_L = Const } $\therefore \bar{E}_\ell = \frac{\rho_L}{2\pi\epsilon} \cdot \frac{\bar{\ell}}{|\ell|^2}$

- $\bar{\ell} = P - N = (2,3,-4) - (2,0,0)$

$= 3\hat{y} - 4\hat{z}$

ملحوظة: لأن الخط المشحون موجود على محور x

لذلك فإن إحداثي ال x للنقطة N هو نفس إحداثي ال x للنقطة P.

- $\bar{R}_1 = (2,3,-4) - (0,0,1) = 2\hat{x} + 3\hat{y} - 5\hat{z}$
- $\bar{R}_2 = 2\hat{x} + 3\hat{y} - 3\hat{z}$
- $|\ell| = \sqrt{9+16} = 5$
- $|R_1| = \sqrt{4+9+25} = \sqrt{38}$
- $|R_2| = \sqrt{22}$

$\therefore \bar{E}_P = \frac{\rho_L}{2\pi\epsilon} \cdot \frac{\bar{\ell}}{|\ell|^2} + \frac{Q_1}{4\pi\epsilon} \cdot \frac{\bar{R}_1}{|R_1|^3} + \frac{Q_2}{4\pi\epsilon} \cdot \frac{\bar{R}_2}{|R_2|^3}$

$= 2\hat{x} + 7.33\hat{y} - 9.38\hat{z}$ V/m.

(b) Find ρ_L so that $\bar{E}\Big|_{(0,0,3)} = 0$

$$\bar{l} = (0,0,3) - (0,0,0) = 3\hat{z}$$

$$\bar{R}_1 = (0,0,3) - (0,0,1) = 2\hat{z}$$

$$\bar{R}_2 = (0,0,3) - (0,0,-1) = 4\hat{z}$$

$$|\bar{l}| = 3 \qquad |\bar{R}_1| = 2 \qquad |\bar{R}_2| = 4$$

$$\therefore \bar{E} = \frac{\rho_L}{2\pi\epsilon} \cdot \frac{3\hat{z}}{9} + \frac{Q}{4\pi\epsilon}\left[\frac{2\hat{z}}{8} + \frac{4\hat{z}}{64}\right] = 0$$

$$\therefore \frac{\rho_L}{6\pi\epsilon} = -\frac{5Q}{64\pi\epsilon}$$

$$\therefore \rho_L = -\frac{15Q}{32} = -3.75 \text{ nC/m}$$

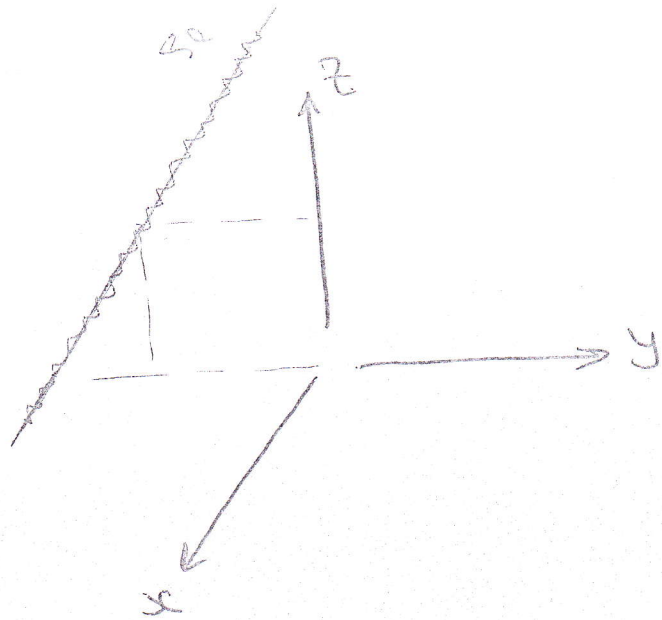
$$\therefore \boxed{\rho_L = -3.75 \text{ nC/m}}$$

مركز النجيب
للخدمات العلمية

[6]

$\rho_\ell = 16\ nC/m$

at $(y=-2, z=5)$



(a) find $\bar{E}$ at $(1,2,3)$.

line $\begin{cases} \text{infinite} \\ \rho_\ell = \text{Const.} \end{cases}$

"الخط موازي لمحور x"

$$\therefore \bar{E}_\ell = \frac{\rho_\ell}{2\pi\epsilon} \cdot \frac{\bar{l}}{|\bar{l}|^2}$$

$$\bar{l} = (1,2,3) - (1,-2,5) = 4\hat{y} - 2\hat{z}$$

$$|\bar{l}| = \sqrt{16+4} = \sqrt{20}$$

$$\therefore \bar{E}_l = \frac{\rho_\ell}{2\pi\epsilon} \cdot \frac{4\hat{y} - 2\hat{z}}{20}$$

$$= 57.5\,\hat{y} - 28.8\,\hat{z}\ (V/m).$$

(b) find $\bar{E}$ at a point on $(z=0)$ xy-plane at which dir of $\bar{E}$ is $\left(\frac{1}{3}\hat{y} - \frac{2}{3}\hat{z}\right)$

$$\bar{l} = (x,y,0) - (x,-3,5) = (y+2)\hat{y} - 5\hat{z}$$

point

$$\therefore \bar{E}_l = \frac{\rho_\ell}{2\pi\epsilon} \cdot \frac{(y+2)\hat{y} - 5\hat{z}}{(y+2)^2 + 25}$$

$\bar{E}$ vector dir

$E_y \longrightarrow \frac{1}{3}$

$E_z \longrightarrow -\frac{2}{3}$

$$\therefore -\frac{2}{3} E_y = \frac{1}{3} E_z$$

$$\therefore E_z = -2 E_y$$

$$\therefore \frac{-5}{(y+2)^2 + 25} = -2 \frac{(y+2)}{(y+2)^2 + 25}$$

$$5 = 2(y+2)$$

$$\therefore y = 0.5$$

$$\therefore \bar{E} = 23 \hat{y} - 46 \hat{z} \quad (V/m)$$

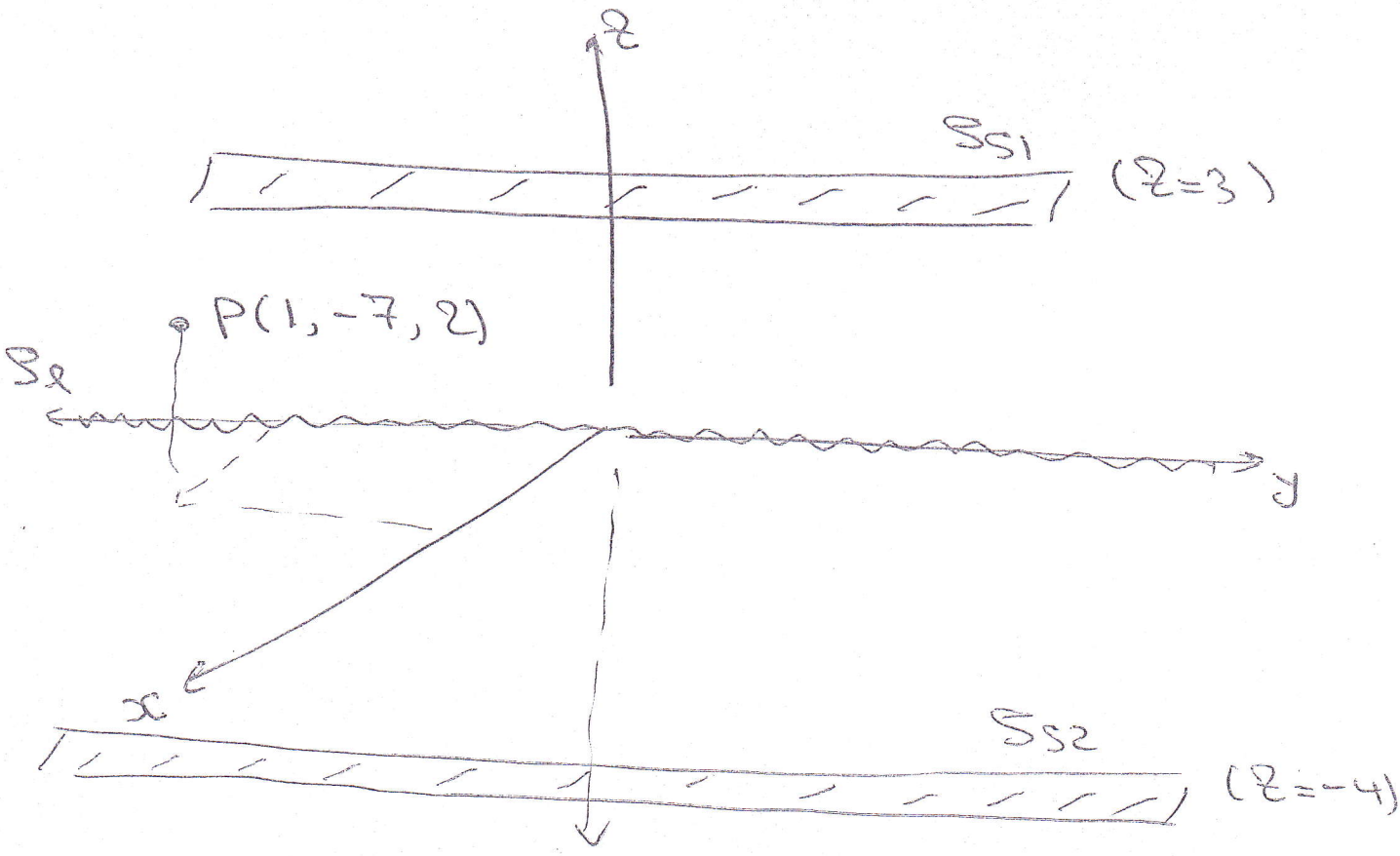
[8] $\rho_l = 2\pi$ nC/m $\longrightarrow$ y-axis

$\rho_{s1} = 0.1$ nC/m^2 $\longrightarrow$ Plane $(z=3)$

$\rho_{s2} = -0.1$ nC/m^2 $\longrightarrow$ Plane $(z=-4)$

Find $\bar{E}_p$ at $P(1,-7,2)$.

Solution



$$\bar{E}_P = \bar{E}_l + \bar{E}_{s1} + \bar{E}_{s2}$$

$\Rightarrow$ Line : infinite and ρ_l = Const.

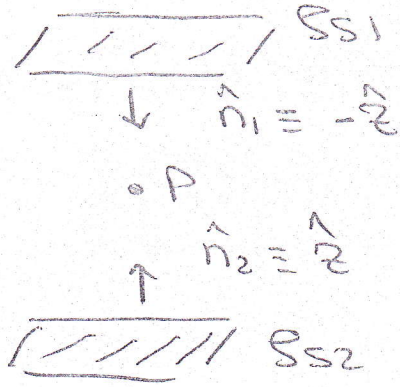
$$\bar{l} = (1,-7,2) - (0,-7,0) = \hat{x} + 2\hat{z}$$

$$\therefore \vec{E}_l = \frac{\rho_l}{2\pi\epsilon} \cdot \frac{\vec{l}}{|\vec{l}|^2} = \frac{\rho_l}{2\pi\epsilon} \cdot \frac{\hat{x} + 2\hat{z}}{5}$$

⇒ Sur 1 and Sur 2 : $\vec{E}_s = \frac{\rho_s}{2\epsilon} \hat{n}$

$$\therefore \vec{E}_{S1} = -\frac{\rho_{S1}}{2\epsilon} \hat{z}$$

$$\vec{E}_{S2} = \frac{\rho_{S2}}{2\epsilon} \hat{z}$$



$$\therefore \vec{E}_P = \vec{E}_l + \vec{E}_{S1} + \vec{E}_{S2}$$

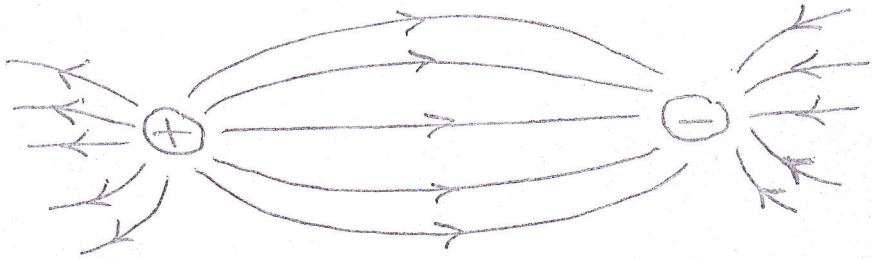
$$= 22.6\hat{x} + 33.9\hat{z} \ (V/m).$$

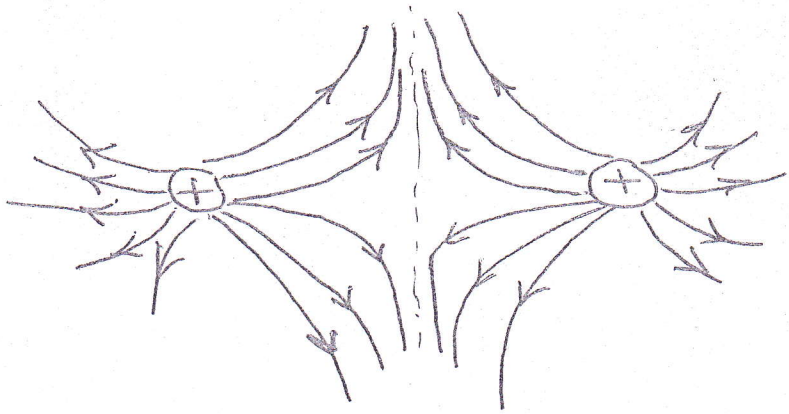
$$\frac{2\pi * 10^{-9}}{2\pi\epsilon}\left(\frac{\hat{x} + 2\hat{z}}{5}\right) + \frac{-0,1*10^{-9}}{2\epsilon_0} * \hat{z} - \frac{0,1*10^{-9}}{2\epsilon_0} * \hat{z} =$$

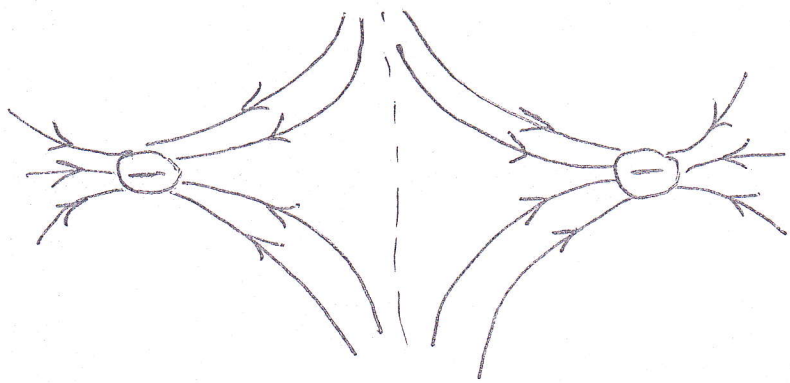
$$22.6\hat{x} + 33.9\hat{z} \ (V/m)$$

Stream Lines

"خطوط الفيض الكهربي"







• خطوط الفيض الكهربي : خطوط وهمية تعبر عن مدى قوة واتجاه المجال الكهربي وهي خطوط لا تتقاطع.

• الشحنة الموجبة : يخرج منها الفيض.

• الشحنة السالبة : يدخل إليها الفيض.

• الشحنات المتشابهة تتنافر والمختلفة تتجاذب.

⇒ Stream lines eqn :

$$\boxed{\frac{h_1\,du_1}{E_{u_1}} = \frac{h_2\,du_2}{E_{u_2}} = \frac{h_3\,du_3}{E_{u_3}}}$$

حفظ

(1) Cart :
$$\frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z}$$

(2) Cyl :
$$\frac{d\rho}{E_\rho} = \frac{\rho\,d\phi}{E_\phi} = \frac{dz}{E_z}$$

(3) Sph :
$$\frac{dr}{E_r} = \frac{r\,d\theta}{E_\theta} = \frac{r\sin\theta\,d\phi}{E_\phi}$$

Ex Find the stream lines eqn of :

$$\bar{E} = \frac{x}{x^2+y^2}\hat{x} + \frac{y}{x^2+y^2}\hat{y}$$

Solution

Cart : $\because E_z = 0$. $\therefore \frac{dx}{E_x} = \frac{dy}{E_y}$

$$\therefore \frac{dx}{\left(\frac{x}{x^2+y^2}\right)} = \frac{dy}{\left(\frac{y}{x^2+y^2}\right)}$$

$$\therefore \frac{dx}{x} = \frac{dy}{y}$$

بتكامل الطرفين

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$\therefore \ln x = \ln y + k$ $\quad k = c_1 c_3$

$\therefore$ eq^n : $\boxed{\ln x = \ln y + k}$

Sheet (2)

[8] $\vec{E} = s^2 \hat{s} - \sec\phi \hat{\phi}$

Find the stream lines eq^n.

Solution

$\Rightarrow$ Cyl : $\dfrac{dp}{E_p} = \dfrac{s\,d\phi}{E_\phi}$

$\therefore \dfrac{dp}{s^2} = \dfrac{-s\,d\phi}{\sec\phi}$

$\therefore \dfrac{dp}{s^3} = -\cos\phi\, d\phi$

$\therefore \int s^{-3}\, dp = -\int \cos\phi\, d\phi$

$\therefore \dfrac{s^{-2}}{-2} = -\sin\phi + k$

$\therefore \boxed{\dfrac{-1}{2s^2} = -\sin\phi + k}$